Joint IMD-WMO group fellowship Training On Numerical Weather Prediction
By
Meteorological Training Institute, India Meteorological Department (IMD), Pune

Atmospheric Boundary Layer (ABL) and its Parameterizations – Part I

Dr. Shivsai Ajit Dixit

Scientist, IITM, Pune

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1. Reference Books

1. Turbulence in the Atmosphere – J. C. Wyngaard (Cambridge University Press) *

2. An Introduction to Boundary Layer Meteorology – R. B. Stull (Kluwer Academic Publishers) *

3. An Introduction to Fluid Dynamics – G. K. Batchelor (Cambridge University Press)

1. Reference Books

4. Boundary-Layer Theory (7^{th} Edition) – H. Schlichting
(McGraw-Hill Book Company)

5. Turbulence – P. A. Davidson
(Oxford University Press)

6. Fluid Mechanics – Kundu and Cohen
(Academic Press)

2. What is a boundary layer?

A region, next to the boundary of the domain of action, not extending over the entire domain, where the effects of the boundary condition(s) - BCs – are felt by the solution of the governing partial differential equations (PDEs).

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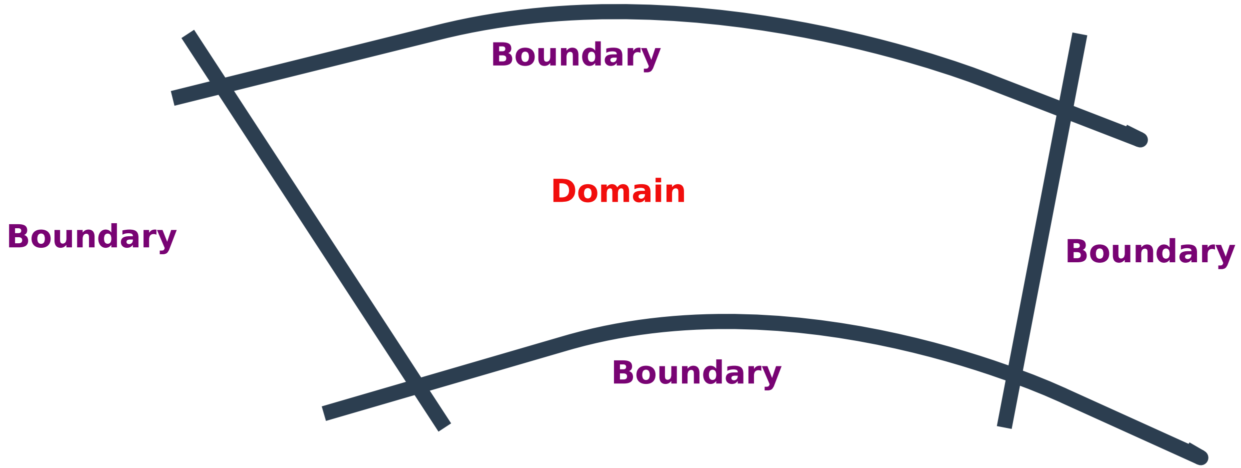

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Purely MATHEMATICAL definition!!

Therefore, there are mathematical boundary layers in the solutions of PDEs!!

2. What is a boundary layer?

Physical boundary layers occur when the PDEs under consideration represent the physical behaviour of substance(s) over a certain region of space (with "boundaries") and over a certain period of time.

Physical boundary layers can occur in phenomena involving deformations of solids, liquids and other media where these deformations or their rates are governed by the PDEs.

3. Fluid Flow and Navier-Stokes Equation

Newton's second law of motion for the flow of fluids takes the form of the "governing" momentum equation

$$\rho \frac{Du_i}{Dt} = \rho \left(\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right) = -\frac{\partial p}{\partial x_i} + \rho g_i + \frac{\partial \sigma_{ij}}{\partial x_j}$$

In a term, subscript indicates three components (say, i=1,2 and 3) and a repeated subscript indicates summation over that subscript

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3. Fluid Flow and Navier-Stokes Equation

For a "Newtonian Fluid", the stress is directly proportional to the strain rate i.e. the "constitutive" relation is

$$\boxed{\sigma_{ij}} = \mu \boxed{\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)} = 2\mu s_{ij}$$

Stress tensor **Strain rate tensor**

3. Fluid Flow and Navier-Stokes Equation

For an "Incompressible Flow", the mass conservation equation reads

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} = 0$$

$$\boxed{\frac{\partial u_i}{\partial x_i}} = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} = 0$$

Velocity field is divergence free!

3. Fluid Flow and Navier-Stokes Equation

Navier-Stokes equation is the momentum equation for the incompressible flow of a newtonian fluid

$$\frac{Du_i}{Dt} = \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j}$$

Body force term has been written as gradient of a scalar potential and has been absorbed in the pressure gradient term

3. Fluid Flow and Navier-Stokes Equation

Note that this is a second-order, nonlinear PDE requiring six BCs (two for each velocity component integration with respect to space) and three ICs (one initial condition for each velocity component integration with respect to time) for its solution

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4. Reynolds Number (Re)

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Inertial term or Advection term (nonlinear)

Viscous term

$$u \sim U \text{ and } x \sim L$$

“Scales”

$$Re = \frac{\text{Inertial}}{\text{viscous}} \sim \frac{U^2/L}{\nu U/L^2} \sim \frac{UL}{\nu}$$

4. Reynolds Number (Re)

$$\frac{D\hat{u}_i}{Dt} = \frac{\partial \hat{u}_i}{\partial \hat{t}} + \hat{u}_j \frac{\partial \hat{u}_i}{\partial \hat{x}_j} = -\frac{\partial \hat{p}}{\partial \hat{x}_i} + \frac{1}{Re} \frac{\partial^2 \hat{u}_i}{\partial \hat{x}_j \partial \hat{x}_j}$$

“Scaled” Navier-Stokes Equation

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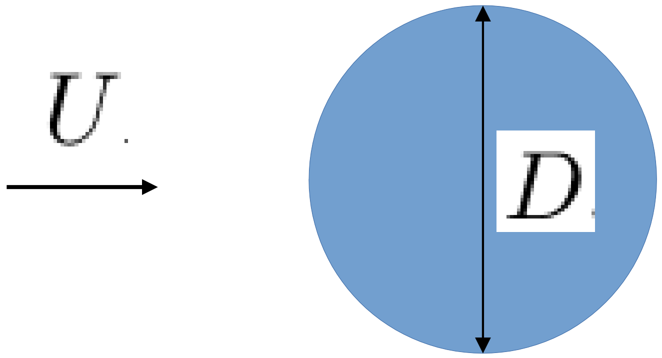

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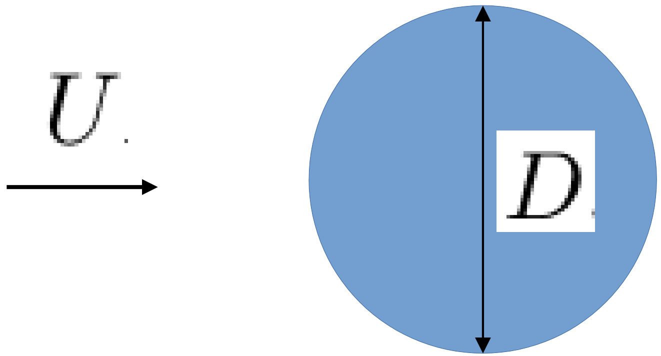

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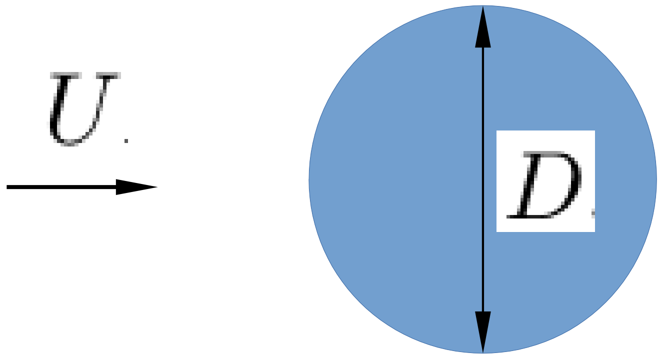
Euler Equation (Inviscid)



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Euler Equation (Inviscid)

In the limit of "infinite" Re, viscous term drops out of the NS equation and one obtains Euler Equation which is inviscid in character.

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Euler Equation (Inviscid)

D'Alembert's Paradox

D'Alembert showed that Euler equation mathematically gives ZERO DRAG force on any body, a prediction that was completely at odds with the experiments.

5. Ludwig Prandtl

Ludwig Prandtl's Resolution of d'Alembert's paradox

- **For fluids of very small viscosity flowing past a solid wall or surface, the velocity of the fluid at the wall must be the same as the velocity of the wall itself i.e. there is NO SLIP between the fluid and the wall. This is the "no slip" or "wall" BC.**
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- **To meet this “no slip” or “wall” BC, the fluid velocity must rapidly go from the “outer” inviscid solution value to zero at the wall over a short wall-normal distance. The layer of fluid over which this happens “knows” about the presence of the wall and is called as the “Boundary Layer”.**

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- **The flow outside the boundary layer is inviscid and does not “know” or “feel” the presence of the wall. This is because the Euler Equation cannot accommodate the “no slip” BC due to loss of the second-order viscous term as a result of “scaling” wall-nomal variations also using the larger length scale.**

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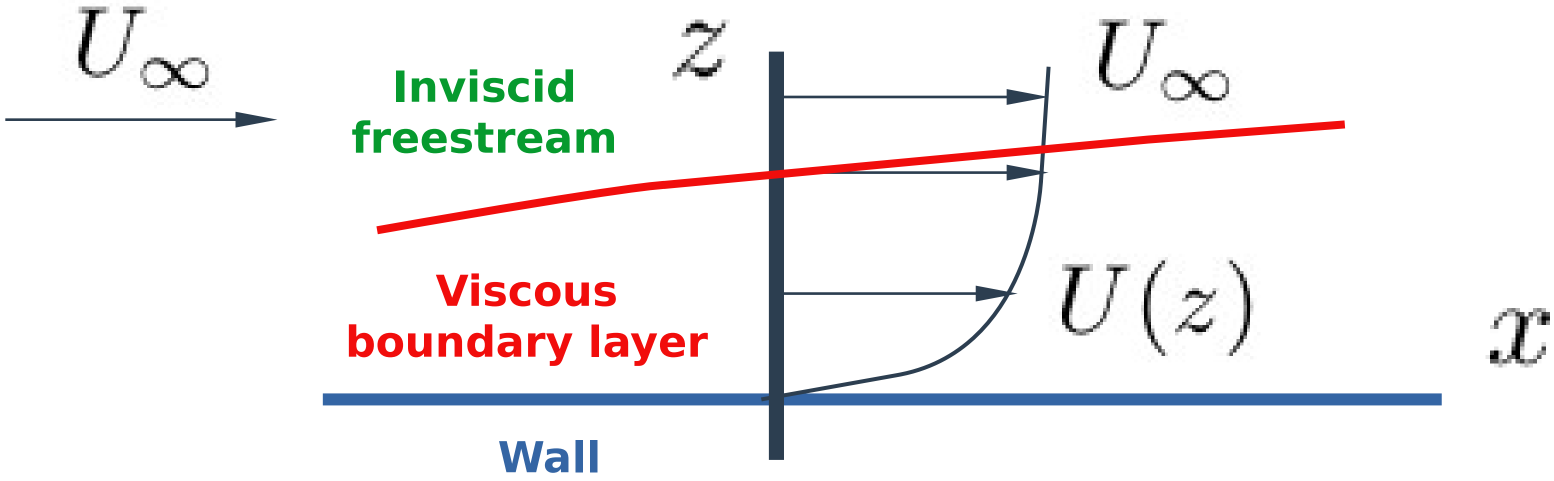
- **This viscous term, however, remains important within the "boundary layer" because the "no-slip" BC must be met. Therefore, "rescaling" of the Navier-Stokes equation (in the wall-normal coordinate, especially) is required to retain this term.**

Ludwig Prandtl's Resolution of d'Alembert's paradox

- **The flow field therefore comprises of two regions. (a) Outer inviscid flow and (b) inner boundary layer flow.**



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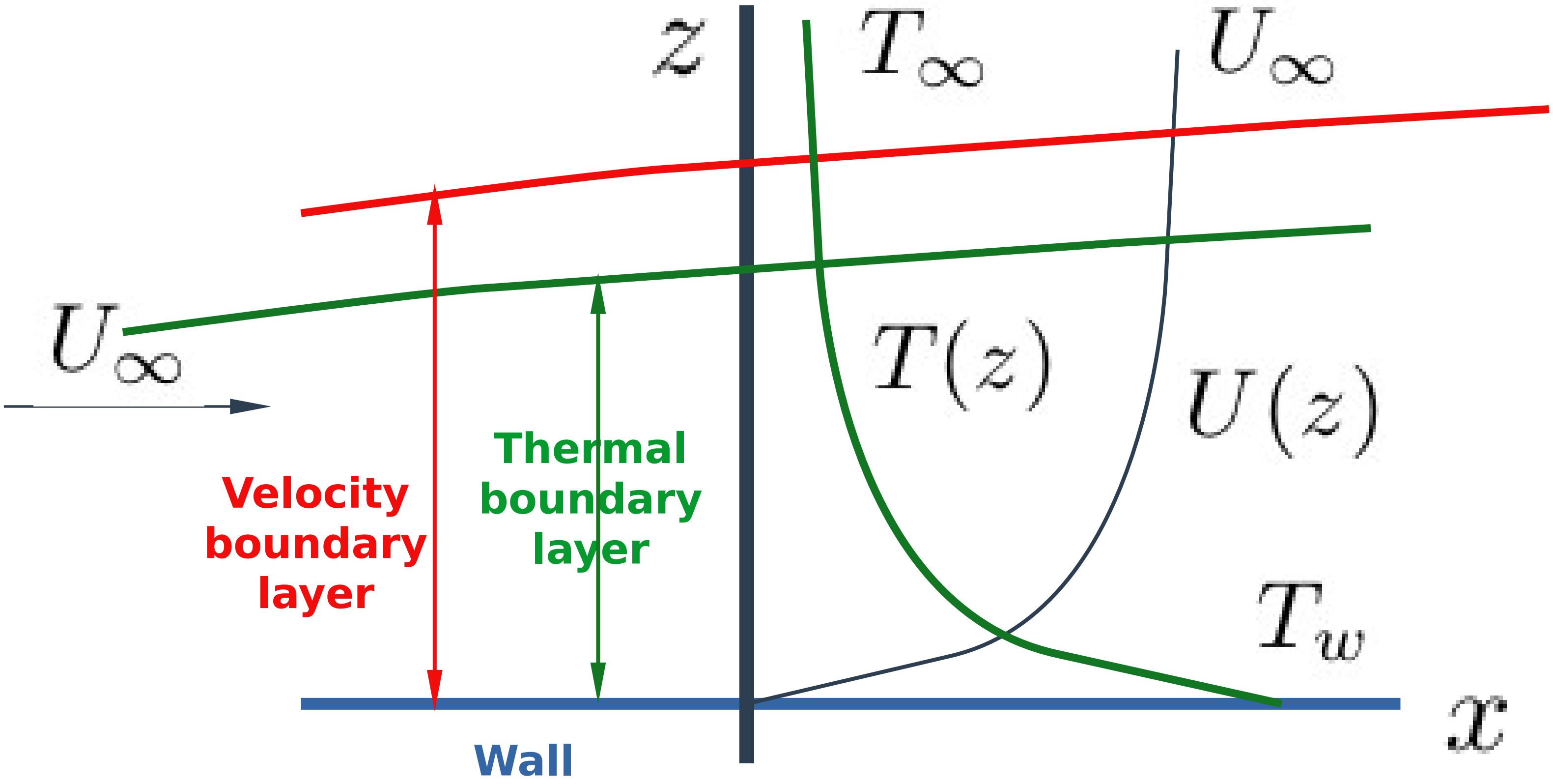

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- Velocity boundary layer thickness depends on the momentum diffusivity
- Laminar flows rely on slow “molecular” diffusion
- **Turbulent flows have eddies that enable very rapid “turbulent” diffusion**